"You Do It" Solution

Lecture 6: Sampling Distributions

Prepared as a classroom-ready detailed solution set for ECOM5005

Exercise 1: Sampling Distribution of Vehicle Tax Values

Question

The mean and standard deviation of the tax value of all vehicles registered in a certain state are

$$\mu = \$13{,}525 \qquad \text{and} \qquad \sigma = \$4{,}180.$$

Suppose random samples of size $n = 100$ are drawn from the population of vehicles. What are the mean $\mu_{\bar{X}}$ and standard deviation $\sigma_{\bar{X}}$ of the sample mean $\bar{X}$?

Detailed Solution

Step 1: Identify the statistic and the relevant sampling distribution.
Let $\bar{X}$ denote the **mean tax value in a random sample of 100 vehicles**. We are not analysing the tax value of one vehicle; we are analysing how the *sample mean* would vary if we repeatedly selected random samples of size 100.
The lecture gives the following two key results for the sampling distribution of the sample mean:

$$\mu_{\bar{X}} = \mu, \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}.$$

Here, $\sigma_{\bar{X}}$ is called the **standard error of the mean**.
Step 2: Calculate the mean of the sampling distribution.
The sample mean is an unbiased estimator of the population mean, so

$$\mu_{\bar{X}} = \mu = \$13{,}525.$$

Therefore,

$$\boxed{\mu_{\bar{X}} = \$13{,}525}.$$

Step 3: Calculate the standard deviation (standard error).
Using $\sigma = \$4{,}180$ and $n = 100$,

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\$4{,}180}{\sqrt{100}} = \frac{\$4{,}180}{10} = \$418.$$

Hence,

$$\boxed{\sigma_{\bar{X}} = \$418}.$$

Exercise 1: Interpretation and Final Answer

Why the Sampling Mean Has the Same Centre

The sampling distribution is centred at the population mean:

$$\mu_{\bar{X}} = \mu.$$

This is why $\bar{X}$ is called an **unbiased estimator** of μ. Across all possible random samples of size 100, the average of the sample means is \$13,525; there is no systematic upward or downward shift simply from taking a sample.

Why the Sampling Means Are Less Variable

The variability of the sample mean is measured by the standard error:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}.$$

For this problem,

$$\frac{\$4{,}180}{\sqrt{100}} = \$418.$$

Averaging many vehicle tax values smooths out some of the vehicle-to-vehicle variation. This is why the distribution of sample means is much narrower than the distribution of individual vehicle values.

Connection to the Central Limit Theorem

Because the sample size is large ($n = 100$), the **Central Limit Theorem** tells us that the sampling distribution of $\bar{X}$ will be approximately normal for most population shapes. Under the lecture's large-population / independent-sampling assumption, we may write approximately

$$\bar{X} \sim N\left(13{,}525,\ 418^2\right).$$

This allows probability calculations involving possible sample means. It also means that, across repeated samples of 100 vehicles, the means fluctuate around \$13,525 with a standard deviation of about \$418.

Answer Summary

Mean of the sampling distribution:

$$\mu_{\bar{X}} = \mu = \boxed{\$13{,}525}.$$

Standard deviation (standard error) of the sampling distribution:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\$4{,}180}{\sqrt{100}} = \boxed{\$418}.$$

Final Check

Remember the distinction:

- $\sigma = \$4{,}180$ describes the spread of **individual vehicle tax values**.
- $\sigma_{\bar{X}} = \$418$ describes the spread of **sample means based on 100 vehicles**.
- Increasing n leaves the centre $\mu_{\bar{X}} = \mu$ unchanged, but decreases the standard error because it is divided by $\sqrt{n}$.